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## Effects of Training in the Use of Social and Cognitive Strategies: An Intervention Study in Secondary Mathematics in Co-Operative Groups\*

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### ABSTRACT

This article reports on an intervention study into the effects of a training in the use of social and cognitive strategies on the learning outcomes of students in secondary mathematics education. Special attention is given to differential effects for high- and low-achieving students. The focus on differential effects is derived from studies into learning in small co-operative groups, and from the results of meta-analyses into the effects of training in learning strategies. From these studies it can be concluded that in general such programs contribute to learning. However, it seems that low-achieving students are unable to benefit from interventions of the kind investigated (i.e., co-operative learning and training in learning strategies). The main question is whether it is possible to design an instructional program from which all students benefit, and from which the low-achieving students profit more than their counterparts in the control-program. In the present study three instructional programs for co-operative learning were compared: (i) an experimental program with special instruction in the use of social strategies; (ii) an experimental program with special instruction in the use of cognitive strategies; and (iii) a control program without training in either cognitive or social strategies. The programs were identical with respect to mathematical content and general instructional settings (a combination of whole-class instruction, working in co-operative groups and individual work). The experiment addressed the following research question: what are the general and differential effects of a training in the use of social and cognitive strategies on the results of learning in secondary mathematics? The research was conducted in two schools for secondary education in a

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total of 21 classes, involving a total of 511 students. The design was a pretest-posttest control group design, using two experimental groups and one control group. The data were analysed from a multi-level perspective. The outcomes of the investigation clearly show the effects of the intervention. Teaching cognitive and social strategies has the expected, positive effects. In addition to this main effect, a compensatory effect for the low-achieving students was found. The low-achieving students in the experimental conditions outperformed their counterparts in the control group.

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## THEORETICAL BACKGROUND

The past twenty years have shown a revival of interest in co-operative learning in many countries. Although there is a long European tradition in co-operative learning and peer teaching, the impetus came from the co-operative movement in the United States (e.g., Wagner, 1982; Webb & Farivar, 1994).

This interest grew because co-operative learning programs promised success for all students. Various researchers reported positive overall effects on learning performances (Fantuzzo, King & Heller, 1992; Meloth & Deering, 1992, 1994; Shachar & Sharan, 1994; Slavin, 1989, 1990). Other studies, however, showed no positive overall effects on learning achievement (Davidson, 1985; Vedder 1985).

In addition to these findings indications were found that high- and low-achieving students benefit differently from learning in small groups. (Leechor, 1988; Webb, 1989, 1991; Webb & Farivar, 1994). These differences in learning gains between high- and low-achieving students may be related to social and cognitive factors. However, not all studies confirm these research findings. In addition, findings across different co-operative learning methods are not consistent. Some methods foster higher achievement with certain kinds of tasks, while others do not (Shachar & Sharan, 1994).

Recently, researchers have taken a closer look at the interaction processes in small groups in order to explain possible differences in effects between high- and low-achieving students (Cohen & Lotan, 1995; Good, Mulryan & McCaslin, 1992; Webb & Farivar, 1994).

There are at least two theoretical orientations in explaining differential effects: sociological and cognitive. These two theories share a main concept, namely 'access to resources' (Cohen & Lotan, 1995; Prawat, 1989).

From *sociological and social-psychological* theories it is conceivable that students within heterogeneous groups have different access to resources. Students in groups develop status orders. These status orders are based on perceived differences in academic success. Status differences

have the effect of depressing the participation of low status students in small group interaction. As a consequence, differences in achievement increase (Cohen & Lotan, 1995). However, it is not only the rate, but also the nature of the participation which influences learning in co-operative groups. Students learn more from receiving elaborated help from other group members and less from receiving low-level elaboration (for example, receiving only the answer to a problem). Low-achievers are not always able to ask for the right help, because it is difficult for them to explain what they do not understand. From the sociological and social-psychological perspective, learning can be promoted by enhancing the rate and nature of participation. This can be achieved by social strategy training and by influencing classroom interaction patterns in a direction which guarantees all students 'equal access' to 'resources', for example by promoting and improving helping behaviour in co-operative groups (Cohen & Lotan, 1995; Webb, 1989, 1991; Webb & Farivar, 1994).

From a *cognitive perspective*, theorists state that low-achieving students benefit less from group work since they lack prior knowledge (declarative, procedural, strategic, metacognitive, and situational). Differences in access to resources are primarily seen in terms of knowledge and strategies. Low-achieving students are not always able to use knowledge and problem-solving strategies (at the right moment). As a consequence of deficiencies in strategic and metacognitive knowledge, low-achievers are not always able to cope with the strategies used by high-achievers. On their part, high-achievers are not always able to explain their routinely used strategies. From the cognitive perspective the promotion of learning in co-operative groups can be realised by improvement of their strategies and metacognitive awareness. This can be done by training students in the use of (meta-) cognitive strategies, for example problem solving strategies and control strategies such as planning, monitoring, checking, and revising. This kind of training is most successful if conducted within a school subject or domain in the context of the school curriculum, rather than in isolation. Promoting the reflection processes is very important, for example by demonstration, practice, feedback and discussion (Ausubel, 1968; Chinnappan & Lawson, 1996; De Corte & Verschaffel, 1988; Prawat, 1989; Resnick, 1989; Schoenfeld, 1985, 1992).

These explanations, from the sociological/social-psychological and the cognitive perspective, are complementary in explaining the differential effects of small-group learning. Various social and cognitive factors may hinder low-achievers to obtain access to the resources and may consequently prevent learning. As a consequence, differences between high- and low-achieving students increase. However, not all research findings

are consistent in this respect and certainly not across different co-operative learning methods (Shachar & Sharan, 1994).

In line with these theoretical perspectives, in empirical studies differential effects of small group learning for high- and low-achieving students in secondary mathematics were found (Terwel & Van den Eeden, 1992a, 1992b). High-, and medium-achieving students seem to benefit more from learning in co-operative groups than low achievers. In a resource-theoretical explanation of these findings the 'threshold hypothesis' of Dar and Resh (1994) was mentioned. This hypothesis states that in order to benefit from a learning environment, a certain minimum of resources ('the threshold') is required. In this perspective the 'Matthew effect' is also conceivable: the strong become stronger since their superior resources place them in a favourable position compared to less resourced students (Dar & Resh, 1994; Terwel & Van den Eeden, 1992a, 1992b; Van den Eeden & Terwel, 1994). The notion of 'resources' makes it possible to look both at the social and the cognitive factors as resources; they can be defined in terms of social strategies or cognitive strategies.

From these theoretical and empirical analyses, ideas emerged for an investigation into the effects of training in social and cognitive strategies. The training should provide more students with access to resources, allowing them to participate effectively in co-operative problem solving. An intervention study was developed in which two different experimental programs and a control program were implemented. In the experimental programs students were trained either in the use of social strategies for small-group participation or in the use of cognitive strategies for solving mathematical problems.

What do we learn from research literature into the effects of training in the use of social or cognitive strategies? In general there is reason to expect positive effects from training in the use of social or cognitive strategies. From empirical research into social strategy training there is some evidence that the effects, in most cases, are positive. Webb and Farivar (1994) reported positive effects of small-group social interaction training for some groups but not for others. There is more evidence from research into cognitive strategy training. In a recent meta-analysis including 51 studies, positive overall effects were found, especially for the type of strategy training in which reflection on the how, when, where, and why of the use of strategies was stimulated. However, researchers report that low-achieving students are unable to benefit from interventions of most kinds. Surprisingly, they also found a few exceptions (Hattie, Biggs, & Purdie, 1996). In an empirical study into the effects of problem solving

training in mathematics Chinnappan and Lawson (1996) also found positive effects on performance of both high-, and low-achieving students.

It may be concluded that there is some evidence regarding the effectiveness of training in the use of social or cognitive strategies. However, there is good reason to examine the problem of differential effects for high-, and low achieving students in greater detail, especially in the context of co-operative group learning. Given the concerns regarding low-achieving students mentioned above, in the present study special attention will be given to the counterbalancing forces of remedial instruction and guidance ('scaffolding') for the low-achieving students. The concept of scaffolding originates from sociocultural (Vygotskian) theories. Scaffolding is closely related to a main concept of sociocultural theory called 'the zone of proximal development'. The metaphor of a scaffold resembles the finely tuned, temporary support that can be removed when no longer needed (Azmitia & Perlmutter, 1989; Brown, & Palinscar, 1989; Collins, Brown, & Newman, 1989; Tudge, 1990).

## RESEARCH QUESTION AND HYPOTHESES

The experiment addresses the following question: what are the general and differential effects of training in the co-operative use of social or cognitive strategies on the results of learning in secondary mathematics? Three hypotheses were formulated.

### **Strategy Training Hypothesis**

Training in the use of social strategies or cognitive strategies has a positive effect on learning outcomes (Chinnappan & Lawson, 1996; Hattie, Biggs, & Purdie, 1996; Webb & Farivar, 1994).

### **Differential Effect Hypothesis**

High- and low-achieving students will benefit differently from co-operative learning. (In operational terms: as a consequence of their superior resources, high-achieving students will gain more per score unit on the pre-test than low-achieving students. Thus, a curvilinear relation between pre-test and post-test is expected in the experimental groups as well as in the control group, because both conditions have co-operative learning.)

### **Remedial Instruction Hypothesis**

Given a context of experimental programs in which special (remedial) instruction and guidance by the teacher is given to the lower-achieving

students, it is expected that low-achievers in the experimental program will benefit more from working in small groups than low achievers in the control program (see also the concept 'scaffolding' in socio-cultural theory: Azmitia & Perlmutter, 1989; Brown & Palinscar, 1989; Collins, Brown, & Newman, 1989).

These hypotheses were tested using a multilevel model. In this model the direct effect of the pre-test score on the post-test score is controlled. In the analyses no dichotomisation between high- and low-achieving students is realised but the student sample remains intact by using continuous variables.

### RESEARCH DESIGN, INSTRUMENTS, AND PROCEDURE

The design for this study is a quasi-experimental pre-test post-test control group design in which three different instructional programs were compared. The sample involved 13 teachers, in 21 classes at two schools, with a total of 511 students. School 1 has a mainly white student population from a middle class background. The school is located in a prosperous village in the centre of the Netherlands. School 2 has a heterogeneous population from Dutch and immigrant background. This school is located in a new, developing town in the centre of the Netherlands. Within both schools student composition in classes is heterogeneous.

For the experiment three different programs were developed: the Social Program (144 students in 6 classes), the Cognitive Program (172 students in 8 classes), and the Control program (195 students in 8 classes). In both schools class composition was unstreamed for the first school year in secondary education. All three programs covered 14 lessons in mathematics in the first year of secondary school (12–13 year olds). In order to eliminate possible school effects, all programs were implemented at both schools. In order to avoid contamination, teachers from different conditions were asked not to communicate with each other about the special features of the program in which they participated.

A mathematical reasoning test was administered prior to, and at the end of the experiment to all participating students. The learning-effect (testing-effect) for students doing the same test before and after the implementation of the programs was assumed to be identical for the three programs. The test consisted of two subscales, nos. 3 and 4 of the *Prüf-system für Schul- und Bildungsberatung* (PSB; Horn, 1969). According to Horn (1969) both scales have a high load on the factor 'reasoning'. In earlier research correlations between PSB subscales 3 and 4 and mathe-



mathematical achievement between .50 and .80 were found (Aurin, 1966; Herfs, Mertens, Perrenet, & Terwel). Scores on the pre-test were used as control variable, with the post-test scores as dependent variable. The total test contained 80 items, with possible score ranges from 0 to 80 correct items. The alpha reliability-coefficients of the pre-test and the post-test are .81 and .75, respectively.

## IMPLEMENTATION OF THE PROGRAMS

All participating teachers were prepared at the start of the experiment by a short training in co-operative learning. In addition, they received a manual in which the instructional model was explained and examples were given. Teachers in the social and cognitive condition respectively received an extra training with opportunity to practice coaching according to their specific condition. The implementation was checked by observation. All students completed a training and received curriculum materials containing the same mathematical content and topics, but with different assignments and guidelines for the experimental groups. How were the programs put into practice?

Because it is beyond the scope of this article to provide a full description of the program and the classroom processes, a short, idealised but typical description of a process-cycle in the cognitive program is given below. *Mutatis mutandis*, the same kind of processes can be described for the social program. A more detailed description of the programs, with samples, is given in Appendix 1.

In terms of Mason and Goods' classification (1993), we can describe the instructional model as a whole-class model that provides for student diversity through small group ad-hoc remediation and enrichment on a daily basis. The whole experiment consisted of a series of 14 lessons about mathematical problems in real-life situations. For theory and examples of authentic mathematical problems in real life situations see Freudenthal (1973), and Terwel (1990).

The cycle begins with whole-class instruction. The aim of this introduction is to motivate students by situating the mathematical content in their daily-life contexts; to allow them to review the necessary pre-knowledge; to give an overview of the learning unit; and to introduce the most important concepts and procedures. In this whole-class instruction the teacher demonstrates strategies for solving mathematical problems. He/she serves as a leader or expert for his/her students in a cognitive apprenticeship model. The methods used involved exploration, coaching, articu-



lation, and reflection. (Collins, Brown, & Newman, 1989; Schoenfeld, 1985, 1992).

For example, the teacher reads the assignment aloud, and asks students how the problem could be represented. He/she stresses the importance of the first steps in the problem-solving process. The process of creating a mental representation of the problem situation is completed by the construction of a schema, diagram, graph, or drawing. In the next step a plan is made in order to solve the problem.

Subsequently the teacher asks what steps can be taken, which procedures are adequate, and whether a heuristic can be used. During and after the execution of the plan, the teacher asks whether the participants are still on the right track. Finally, teacher and students look back, reflect on the process, and check the answer (Schoenfeld, 1985, 1992; Van Streun, 1989, 1994).

Students subsequently start working in small co-operative heterogeneous groups of students on course materials with mathematical problems; and for the experimental groups special assignments in strategic learning. In this stage, the teacher has the opportunity to observe interaction processes. Where necessary, the teacher intervenes in the interaction process in order to assist students in putting the cognitive strategies into practice (practice and feedback). After the work in small heterogeneous groups and after a diagnostic test the outcomes are discussed in a whole class session.

The students then work individually in small homogeneous groups of 4 students (temporary 'within class setting', i.e., low-, medium-, and high-achieving groups). The group of low-achievers works as a remedial group under the direct guidance and supervision of the teacher. Meanwhile the medium-, and high-achieving students work on enrichment tasks at the appropriate level.

In the next stage all students work individually in the context of their own level group, with the opportunity of helping each other. Here too if needed, the low-achievers receive adapted guidance from their teacher. At the end of the session the teacher winds the teaching learning cycle down with a recapitulation of the most important concepts, procedures and strategies. Finally, the post-test is administered, after which teacher and students look back on the whole cycle and reflect on the process and the results of learning.

## RESULTS AT INDIVIDUAL STUDENT LEVEL

In Table 1 the characteristics of the distributions of the pre-test and post-test are presented. In the process from pre-test to post-test all groups show learning gains, but the experimental groups gain more than the control group. The gain scores (difference between the post-test and pre-test) in the control, social, and cognitive groups: 1.57, 3.18 and 4.41 respectively.

In a one-way analysis of variance no significant differences on the pre-test scores between the three groups were found ( $F(2,508) = .23; p = .80$ ). A one-way analysis of variance resulted in significant differences between the post-test scores ( $F(2,508) = 9.59; p = .0001$ ). The analysis of covariance with pre-test as a covariant also shows significant results ( $F(3,507) = 109.79; p = .000$ ). Thus it can be concluded that in general there is a positive effect of the experimental programs on learning results. This result is in keeping with the 'strategy training hypothesis'. The effect size, as defined by Cohen (1988), is .32 for the social program, which is a small effect. The effect size for the cognitive program is .51, a medium effect.

Table 1. Characteristics of the Distributions of the Mathematical Reasoning Pre-test and Post-test for All Students.

	Mean	SD	Min	Max.
Control program <sup>1</sup>				
Pre-test	53.08	6.31	29.0	67.0
Post-test	54.65	7.91	21.0	70.0
Difference	1.57			
Social program <sup>2</sup>				
Pre-test	53.49	6.75	36.0	69.0
Post-test	56.67	6.83	37.0	71.0
Difference	3.18			
Cognitive program <sup>3</sup>				
Pre-test	53.49	6.45	30.0	70.0
Post-test	57.90	6.59	36.0	72.0
Difference	4.41			

Note. *N*-students = 511, *N*-classes = 21.

<sup>1</sup> *N*-students = 195 *N*-classes = 8

<sup>2</sup> *N*-students = 144 *N*-classes = 6

<sup>3</sup> *N*-students = 172 *N*-classes = 8

Table 2. Characteristics of the Regression of the Post-Test on the Pre-Test.

	a (intercept)	b (slope)	$R^2$	$N$
Control program	13.40	.78 (.08)	.38	195
Social program	22.71	.64 (.07)	.36	144
Cognitive program	28.47	.55 (.05)	.39	172

*Note.* Standard deviation indicated between parentheses.  $N$ -students = 511,  $N$ -classes = 21.

In order to get a first impression of differential effects for high- and low achieving students, the characteristics of the intercepts and the slopes of the regression lines in the three conditions are presented in Table 2.

The control group shows a relatively low intercept (13.40) and a relatively steep slope (.78). In the cognitive group the reverse can be seen: a relatively high intercept (28.47) and a less steep slope (.55). The social group occupies an intermediate position. The first impression from these data is that most students profit from training in social and cognitive strategies but that low achievers profit most. In Figure 1 the regressions from Table 2 are presented in a graph.

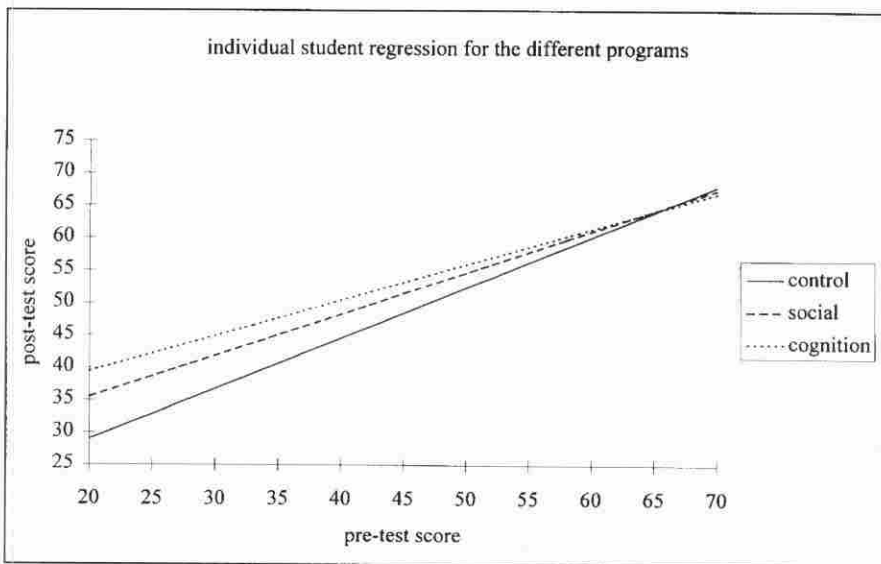


Fig.1. Regression lines of the post-test on the pre-test at individual level.

The next section deals with the multilevel analysis to search for more detailed confirmation of our hypotheses.

## MODEL AND RESULTS OF MULTILEVEL ANALYSIS

To assess the effects of the programs regarding varying degrees of achievements, the appropriate tool is a multilevel model of analysis (Terwel & Van den Eeden, 1992a, 1992b; Van den Eeden & Terwel, 1994). The statistical program ML3-E was used in the analysis (Prosser, Rasbash, & Goldstein, 1993). A multilevel analysis with a continuous variable as indicator of high- and low-achieving students is superior to a regression analysis or multilevel analysis in which separate categories for low-, mediate-, and high-achieving students are used. An analysis with continuous variables is more efficient and the resulting estimates are more accurate.

The multilevel analysis was directed at the relation between achievement in mathematical reasoning after completing the program (the post-test) and variables at the individual student level (pre-test) and the class level (program). Within the analysis model these relations will be described in terms of dependent variable regressions (mathematical reasoning) on the independent variables at student and class levels.

The model basically consists of two steps: the first step concerns a within-group regression, in the second step the results of the first step are introduced in a between-group regression. The two steps of the model can be formulated as follows, with the classroom taken as group level. The student-level regression of the first step is expressed in the following equation:

$$\text{Posttest}_{ij} = \beta_{0j} + \beta_{1j} \text{Pretest}_{ij} + \beta_{2j} \text{Pretest}_{ij}^2 + e_{ij} \quad (1)$$

where

- i: individual student ( $i = 1, \dots, 511$ )
- j: class ( $j = 1, \dots, 21$ )
- m: independent variable at student level ( $m = 1, \dots, 2$ )
- $\beta_{mj}$ : slope of regression of post-test on variable m of class j
- $\beta_{0j}$ : intercept of class j
- $e_{ij}$ : disturbance term, with mean 0 and variance  $s^2$
- $\text{Pretest}_{ij}$ : score on mathematical reasoning test at the beginning of the program

- Posttest<sub>ij</sub>: score on mathematical reasoning test after completing the program
- Pretest<sub>ij</sub><sup>2</sup>: squared score on mathematical reasoning test at the beginning of the program

The second step concerning the regressions between classes of the intercepts  $\beta_{0j}$  and the slopes  $\beta_{mj}$  on the class variable (in casu the program variable).

$$\begin{aligned}\beta_{0j} &= \gamma_{00} + \gamma_{01} \text{Program}j + u_{0j} \\ \beta_{mj} &= \gamma_{m0} + \gamma_{m1} \text{Program}j + u_{mj}\end{aligned}\quad (2)$$

where

- $\gamma_{00}$ : intercept of regression of  $\beta_{0j}$  on class variable Program
- $\gamma_{mn}$ : slope of regression of  $\beta_{mj}$  on class variable Program ( $n = 1$ )
- $u_{0j}$ : disturbance term of regression of  $\beta_{0j}$ , with mean 0 and variance  $\sigma_{0m}^2$
- $u_{mj}$ : disturbance term of the regression of  $\beta_{mj}$ , with mean 0 and variance  $\sigma_m^2$

The model contains the between-class variance, to explain occasional differences in intercepts and slopes. The part of the model that contains the intercept- and slope coefficients of specific variables is the 'fixed part' of the model; the part that contains the disturbance terms is the 'random part'.

With reference to model 1, the variance decomposition of the dependent variable over the two levels is estimated first. Subsequently, the student variables are introduced in the fixed part of the model and the differences in the regressions between the classes are estimated, to allow explanation by class variables. The last analysis is described by model 2. The results of the multilevel analysis are presented in Table 3.

In Table 3 two models are presented. In model 1 the total variance of the post-test is decomposed into a within-class part and a between-class part, indicated by  $s^2$  and  $t^2$  respectively. The within-class variance is 46.3 or 86 per cent. The between-class variance is 7.31 or 14 per cent. This small proportion between-class variance indicates that classes are unstreamed to a large extent. In model 2 the effect of the pre-test and the differences in student aptitude (indicated by pre-test squared) are introduced. Moreover, the two conditions, as compared to the control condition, are incorporated into the model. It turns out that the differential effect is positive (.02), meaning that the stronger students are, the more

Table 3. Results of the Multilevel Analysis, in which the Score on the Mathematical Reasoning Ability Post-Test is the Dependent Variable.

	Model 1	Model 2
Fixed part		
Pre-test		-1.05 (.38)
Pre-test-squared		.02 (.004)
explaining between-class slope differences pre-test by		
SOCIAL program		.23 (.11)
COGNITIVE program		.38 (.10)
explaining between-class slope differences pre-test-squared by		
SOCIAL program		-.004 (.0019)
COGNITIVE program		-.006 (.0017)
Random part		
s <sup>2</sup> (student)	46.3 (2.96)	28.29 (1.81)
class:		
v <sub>1</sub> <sup>2</sup> (intercept)	7.31 (2.83)	.0 (.0)
t <sub>1</sub> <sup>2</sup> (pre-test)		.0009 (.0004)
t <sub>2</sub> <sup>2</sup> (pre-test-squared)		.0 (.0)
Model statistics		
Likelihood ratio	3443.48	3183.67
Difference		259.81
Difference degrees of freedom		6
Difference with model		2

*Note.* Standard deviation between parenthesis, *N*-students = 511, *N*-classes = 21. Non-significant effects are omitted.

they generally gain in learning. In addition, positive effects of the social and cognitive condition were found: .23 and .38 respectively. There is a negative effect of the interaction of the differential effect and the social and cognitive condition: the coefficients are -.004 and -.006 respectively. This outcome means that the stronger students are, the less they profit from the social and cognitive conditions. The sign of the effect from pre-test to post-test is negative (-1.05). The outcomes of the multilevel analysis in Table 3, model 2, are represented in the graph in Figure 2.

Both the cognitive program and the social program lead to increased scores on the post-test for most of the students, especially for the low- and medium achievers. The effect of the cognitive program is stronger than that of the social program. The intersection of the corresponding regression line and the regression line of the control group is situated on a

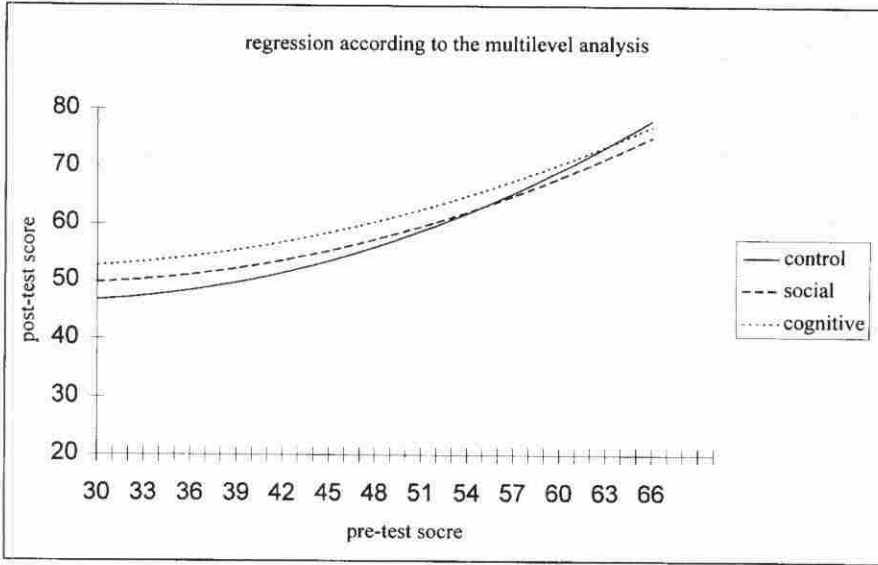


Fig. 2. Regressions of the post-test on the pre-test, according to the multilevel analysis.

higher level than that of the social program. The low- and medium-achieving students benefit especially from the experimental programs, while the outstanding students seem to face a small loss. However, this loss does not by any means outweigh the gains experienced by the weak and medium students. The net effects are clearly positive.

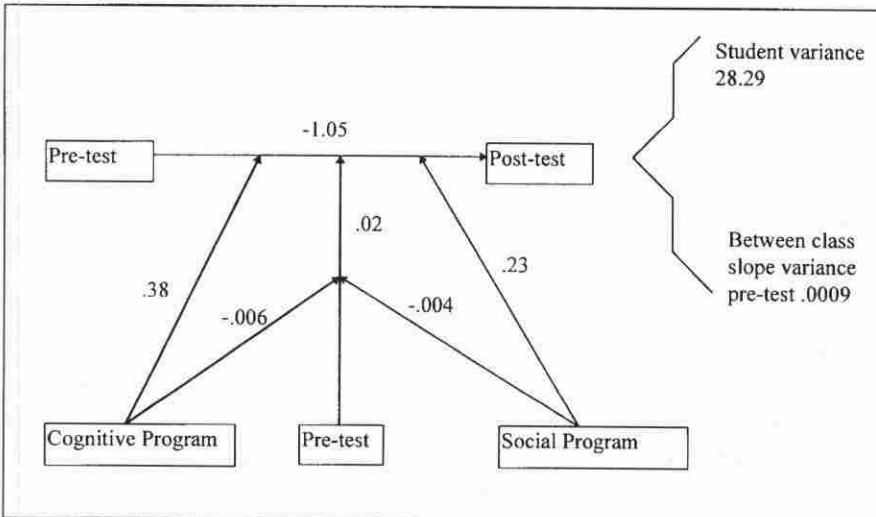


Fig. 3. Diagram of the multilevel analysis.



In Figure 3 the results of the multilevel analysis are presented in a diagram in which the direct effects and the interaction effects are presented.

An elaboration of the outcomes of the previous multilevel analysis can now be given. The between-class differences among the effects of pre-test depend positively on the experimental conditions. The coefficient of the social program is somewhat lower than the coefficient of the cognitive program (.23 and .38 respectively). Moreover, the learning gain is greater under the conditions mentioned than under the control condition. The learning process is the fastest in classes subjected to the cognitive condition. By contrast, the between-class differences due to pre-test-squared depend negatively on both conditions. Again, the coefficient of the cognitive condition is higher than the coefficient of the social condition ( $-.004$  and  $-.006$ , respectively). In interpreting these figures it has to be borne in mind that they concern unstandardised coefficients, and that, as a result, the effects are stronger than they appear to be on the surface. The learning process, which accelerates forwardly, is mitigated by a retardation effect caused by the conditions. The stronger students are, the more they gain (in line with the Matthew effect, mentioned above), but under the social and cognitive conditions the gain of the very high-achieving students is less than under the control condition. Consequently, the experimental conditions result in a tendency to homogenisation.

## CONCLUSIONS, DISCUSSION AND A LOOK AHEAD

The central research question in this article is: what are the effects of co-operative group training in the use of either social or cognitive strategies on the learning results of high- and low-achieving students in mathematical reasoning? Given the initial hypotheses and the analyses presented, three conclusions can be formulated.

First, training in cognitive as well as social strategies had a positive main effect on mathematical reasoning of students in secondary mathematics.

Second, the 'differential effect hypothesis' was also confirmed. This means that high- and low-achieving students benefited differently from co-operative learning in all programs. As a consequence of their superior resources, high-achieving students gained more per score unit on the pre-test than low-ability students. A curvilinear relation between pre-test and post-test occurred in the experimental groups as well as in the control group.

Third, low-achievers in the experimental programs gained more in comparison with the low-achievers in the control program. The special (reme-

dial) instruction given to the low-achieving students had a positive effect. Thus the 'remedial instruction hypothesis' is confirmed. As a consequence the general tendency implied by the differential hypothesis ('the rich are getting richer' or the 'Matthew-effect') was mitigated. To put it differently, the low-achieving students profit more from co-operative learning when given finely tuned support in the use of either cognitive or social strategies (scaffolding).

The outcomes of this study clearly show the benefits of teaching students how to use strategies in solving mathematical problems and to work effectively in small, co-operative groups. Although, generally speaking, the outcomes of this experiment are in line with the research on strategy training, from literature it is known that low-achieving students do not always benefit from this kind of intervention (Hattie, Biggs, & Purdie, 1996).

The differential effects of learning in co-operative groups can, at least to some extent, be mitigated by training in the use of strategies. Depending on one's philosophy of education, this 'mitigating effect' may be seen as satisfactory or unsatisfactory. In any event, there could be some concern about a small negative effect on the small group of high achieving students. Our philosophy is that all students should profit. The fact that the low-achieving students profit should not imply that a few bright students lose out. So there is room for more research into differential and compensatory effects of instructional designs for learning in small co-operative learning groups. In order to be able to separate more clearly the differential effects from the remedial effects, it is recommended to create a fourth condition in the experiment, alongside the social, cognitive and control conditions.

The question of implementation of the programs is not addressed in this article. However, in the present study there are research data about classroom processes from different sources: (i) a student questionnaire concerning the perception of the 'curriculum-in-action' before and after interventions; the Perception of the Curriculum in Action scale (PER-CIA-scale, a learning environment scale); (ii) protocols of classroom and small group processes from qualitative observations; and (iii) information from teacher questionnaires and interviews. These data will be reported in a separate article. The preliminary results suggest that the implementations of the three programs differ significantly in the intended direction, observation supports this conclusion. Especially the classroom processes in the cognitive condition are – in the eyes of the students – different from the processes in the control group. These differences concern instruction by the teacher and quality of co-operation in small groups

of students (clarity of instruction, explanation, strategic learning, reflection, articulation). There is, consequently, some evidence that the positive outcomes of the experiment can be attributed to real differences in classroom teaching and learning processes.

The post-test was a test in mathematical reasoning. Although this test correlates with mathematical achievement (between .50 – .80, as was found in earlier research) this test does not measure domain-specific knowledge. This has the advantage that no 'teaching to test' could occur and the learning results can be considered as 'far transfer'.

It is recommended that a domain-specific test be included in the evaluation. In addition, a test for the assessment of the duration of the effects would be important.

Splitting the instruction into social and cognitive strategies to some extent comes across as artificial to students. Students in the social program asked their group members for explanation of the used procedures. In the cognitive program the students sometimes asked other group members if they understood the given answer. In the present study the social and cognitive programs were designed and implemented separately. However, both programs can be seen as complementary to each other. If the focus is on both problem solving and social strategies, this can result in an even stronger effect. In the near future an experiment combining both strategies will be developed and tested in a field experiment about the additional effects of combining social and cognitive strategies.

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## APPENDIX 1

**Basic Instructional Design**

The AGO model was used as a point of departure in the design of the instructional programs. This instructional model combines aspects of co-operative learning and adaptive instruction. AGO is a Dutch acronym for 'adaptive instruction and co-operative learning'. The model is based on theories about co-operative learning and cognitive learning theory. The AGO model is designed for the middle grades, 12 to 16 year-old students (Freudenthal, 1973; Terwel, 1990; Terwel, Herfs, Mertens, & Perrenet, 1994). In terms of a classification of Mason and Good (1993), we can describe the model as a whole-class model that allows for student diversity through small-group ad-hoc remediation and enrichment on a daily basis. The AGO-model consists of the following stages:

- (1) Whole-class introduction of a mathematics topic in real-life contexts;
- (2) Small-group co-operation in heterogeneous groups of four students;
- (3) Teacher assessments: diagnostic test and observations;
- (4) Alternative learning tracks depending on assessments. These tracks consist of two different modes of activity:
  - 4a. Individual work at own pace and level (enrichment) in heterogeneous groups with the possibility of consulting other students;
  - 4b. Opportunity to work in a remedial group (scaffolding) under direct guidance and supervision of the teacher;
- (5) Individual work at own level in heterogeneous groups with possibilities for students to help each other;
- (6) Whole-class reflection and evaluation of the topic;
- (7) Final test.

The AGO model was the basic instructional design for the two experimental programs and the control program; however some modifications were made, especially in stages 4 and 5. In the present research project heterogeneity was abandoned, particularly with reference to the cognitive program stages 4 and 5, and replaced by homogeneous small groups. This modification was advocated by the teachers who, for practical reasons, preferred a kind of within-class ability grouping (within-class setting with low- medium- and high-achieving students) in stages 4 and 5. Thus rearranging students was no longer required during these stages. In addition, low-achieving students can be located more easily by the teacher because they are seated in the same small ability group.

In the control program the grouping practice was the same. However, there was no special (remedial) instruction or guidance for students in the low-achieving group.

## APPENDIX 2 THE INSTRUCTIONAL INTERVENTION

Before the start of the experiment all teachers (experimental and control group) received the necessary information and a short training regarding (i) the basic ideas and general instructional design; (ii) the curriculum materials; and (iii) working with small co-operative groups. In addition all teachers were asked to read a teacher's manual and to study the curriculum material for the students. The mathematical content was part of the regular curriculum. The booklets were developed by the researchers in co-operation with the teachers.

The intervention in the two experimental conditions consisted of four elements as listed below. All of these were ultimately directed toward the development and use of social or cognitive strategies by the students.

- (A) Specific training for the experimental teachers in the development and use of either cognitive or social strategies.
- (B) Specific training for experimental students in either cognitive or social strategies.
- (C) Additional, specific indications, assignments and hints in the curriculum materials (booklets for the students) regarding either social or cognitive strategies.
- (D) Implementation of a special instructional strategy to promote the development and use of either cognitive or social strategies by the students during the program-in-action, with remedial instruction for low-achieving students.

### **A: Specific Training for the Experimental Teachers in the Development and Use of either Cognitive or Social Strategies**

The teachers, both in the social or cognitive programs, received extra training in teaching students respectively (i) small-group communicating and helping skills, or (ii) cognitive strategies and procedures for solving mathematical problems. Both additional training sessions lasted one afternoon. To give an impression of the cognitive training, one of the assignments under discussion in the cognitive training session is presented in Figure 4.

Teachers experienced, for example, how important it is to construct an adequate representation of the problem and how to reflect on the outcomes. In the training a model for solving mathematical problems was presented, discussed and exemplified in student assignments, taken from the booklets.

During the experiment one of the researchers visited each school for observation and consultation. In addition to this direct form of consultation one of the researchers gave on-line consultations by telephone.



You are working with a power saw and wish to cut a wooden cube, 3 inches on each side into 27 1-inch cubes. You can do this by making six cuts through the cube, keeping the pieces together in the cube shape (see figure beneath). Can you reduce the number of necessary cuts by rearranging the pieces after each cut?

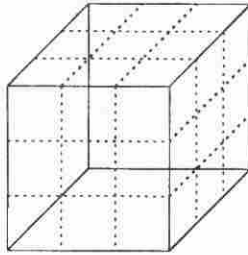


Fig. 4. Sample teacher training: cutting a wooden cube.

### **B: Specific Training for Experimental Students in Cognitive or Social Strategies**

At the start of the mathematical program teachers prepared the students for the program. Students in the control program received a short preparation in small-group co-operation. However, no systematic instruction in either cognitive or social strategies was given to students in the control condition. The teachers in the experimental programs trained their students in using either social or cognitive strategies. To give an impression of the kind of assignments for students in the training a sample from the cognitive training is given in Figure 5.

It was made clear how important it is to make an adequate representation of the problem and to reflect on the outcomes. Students were acquainted with the use of a problem solving model and with processes such as orientation, planning, monitoring, checking and revising. The model for solving mathematical problems was presented, discussed and exemplified by student assignments more or less similar to the ones in their booklets.

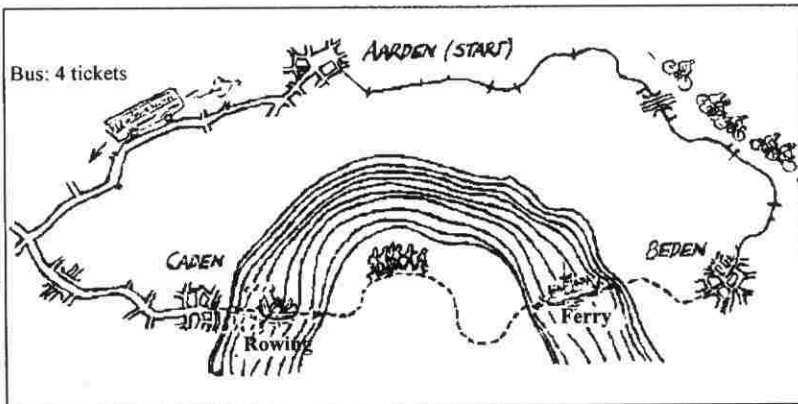
After discussion and reflection students made a poster on which the problem solving process was depicted and described in student language. The poster was fastened on the wall.

During the last week of the school year four first grades groups of students organise a cycle-, walking-, rowing and bus tour.

The number of the students per class are: 1A 28, 1B 30, 1C 27 and 1D 29 students.

In the figure that follows the route is drawn.

Class 1B and 1C cycle, on their own bicycles from Aarden to Beden. From Beden they walk to the ferryboat to cross the river. On the island they also have to go on foot. On the other side the rowing boats, brought by the students of the other classes, are waiting. They row to Caden. From Caden they take the bus back to Aarden. The two other cover the same route the other way around.



The costs are:

Ferryboat per person	F 1.45
Rowboat 6 persons per day	F 45.-
1 bus ticket for 15 zones	F 11.20
Ice cream per student	F 0.90
Soda pop	F 1.50

What are the costs of this touring day per person?

Fig. 5. Problem for the cognitive student training.

Part of the training for the social program was the 'Broken Circles Problem' and the 'Master Designer Problem' (Cohen, 1986). The 'Broken Circles' problem, from the course in social strategies, was given next. The following text is summarised from Cohen (1986, pp. 159 – 164). For the Dutch version we used the text of Stanford (1980). For the 'Broken Circles' problem the class is divided into groups of 3 to 6 students. Each student is given an envelope with different pieces of the circle. The goal is for each person to put together a complete circle. In order for this goal to be achieved, there must be some exchange of pieces. Players are not allowed to talk or to take pieces from someone else's envelope. They are only allowed to give.

When all groups have completed the task or the allotted time has elapsed, the teacher helps the participants identify some of the important things that happened, analyses why they happened, and generalises to other group learning situations. The following questions may serve as a guide to the discussions: What do you think this game was all about? How do you feel about what happened in your group today? What things did you do in your group that helped you to be successful in solving the problem? What things did you do that made it harder? What could the group do better in the future?

The 'master designer' game requires a set of geometric shapes. Each player needs a complete set, but one person in each group takes the role of observer without requiring a set. A total of five students per group is recommended, but smaller groups are acceptable.

One person plays the role of the master designer. This person has to instruct the other players as to how to replicate a design he or she has created with the pieces (all or part of them), but the master designer cannot perform this task for them. Players cannot see what the others are doing, nor can they see the master's design. However, group members may ask the master designer questions. This illustrates an important new behaviour. The group is dependent on the master designer for explaining how it should be done. In addition to verbal directions, children may use sign language to signal to each other. This will help bridge any language differences you may have in your class. When any member of the group feels that he or she has figured out the master design, the designer is asked to check the solution. If the master designer says it is correct, then that player too has to help others in the group. When the play has been completed or the allotted time has elapsed, the teacher, as in the other game, helps the participants to reflect on the process.

With reference to the experiment, after working in small groups on each 'problem' the teacher organised a whole class discussion. During this discussion the students had to tell the significance of the problem and what they had learned. At the end of the student training students were given rules for communication, practice, and feedback concerning small group co-operation. These rules were put on a big poster which was fixed on the classroom wall.

### **C: Additional, Specific Indications, Assignments, Guidelines and Hints in the Program-on-Paper (Curriculum Material/Booklets for the Students/Teacher Manuals) Regarding Social or Cognitive Strategies**

The program-on-paper for the students consisted of a series of lessons together with a specific manual for the teacher. This program is designed on the basis of the general model as described earlier. The lesson materials for the students are enriched with indications, questions and hints to promote the development and use of social or cognitive strategies.

All teachers received a teacher's manual concerning working in small groups, which describes the basic instructional model and some theoretical information concerning small group work. The teachers in the cognitive program received extra manual information on problem solving. The teachers in the social program received additional information on social strategies and communication skills.

In developing the social program we used ideas from different sources and built on experiences from earlier projects that were inspired by the ideas of Cohen, Lotan, and Leechor (1989), Cohen (1986, 1994), Webb (1982, 1989, 1991) and Webb and Farivar (1994).

The cognitive program is based on ideas and research by Polya (1957), Resnick (1989), Riemersma (1991), Schoenfeld (1992), and Van Streun (1989). The students in this program developed and used strategies for mathematical problem solving, with the main objective to develop strategies necessary for problem solving. In our experiment the problem-solving model of Van Streun (1989) was used, with one additional stage. Figure 6 represents the problem-solving model used in the cognitive program.

The ideal typical problem solving process can be described as follows (Van Streun, 1989, 1994). While reading a problem the student first tries to understand the problem. When the student understands the problem he can use three different paths for solving the problem: after understanding the problem the student recognises the problem type and immediately knows the solution, or he can retrieve a problem-solving schema from long-term memory, or he can solve the problem by means of an algorithm

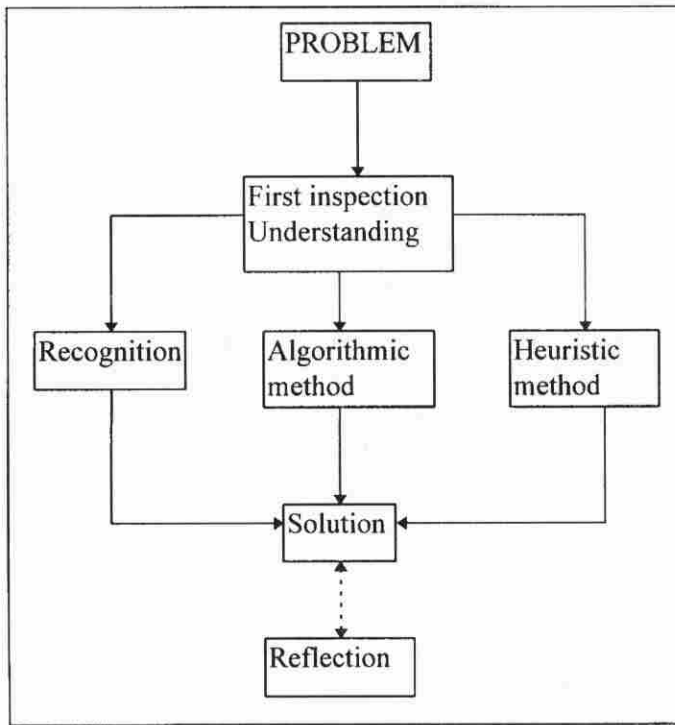


Fig. 6. Problem-solving model used in the cognitive program.

or heuristic. In the reflection stage, the student checks and discusses the answer while keeping the end-goal in mind.

**D: Implementation of a Special Instructional Strategy Aimed at Promoting the Development and Use of Cognitive or Social Strategies by the Students during the Program-in-Action, with Special (Remedial) Instruction for the Low-Achieving Students**

Here we are at the very heart of the intervention, since it is the implementation processes that induces learning (also see the section Implementation of the programs). All students received curriculum material about the same mathematical topic but with different assignments for the experimental groups. Discussing and elaborating the problem from different perspectives is promoted in the cognitive program. The students in the social program are encouraged to give explanations to each other and ask each other for help.